The nth Term for a Sum of Consecutive Triangular Numbers

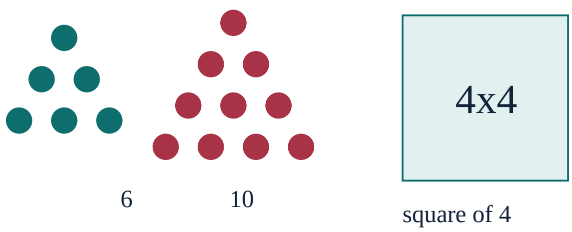
Unit: Number Play - Student Practice Worksheet

Overview: Pixel had already shown that 6 and 10 — two consecutive triangular numbers — add up to 16, a perfect square. Kabir wanted to know: which square, for which pair, without checking every single one by hand. Anaya wrote a single short formula that answered it for every pair, all at once.

Practice Questions (30 Total)

Section A - Easy

Q1. The dot triangles and the 4-by-4 square show 6 + 10. What is that sum?



- [ ] **A.** 16
- [ ] **B.** 14
- [ ] **C.** 18
- [ ] **D.** 20

Q2. The sum of the nth and (n+1)th triangular numbers always equals what?

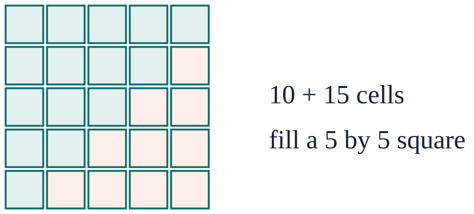
- [ ] **A.** n squared
- [ ] **B.** 2n
- [ ] **C.** n x (n+1)
- [ ] **D.** (n+1) squared

Q3. What is the 5th triangular number?

- [ ] **A.** 10
- [ ] **B.** 15
- [ ] **C.** 21

- [ ] **D.** 12

Q4. The 5-by-5 grid is filled by the 4th and 5th triangular numbers (10 and 15). How many cells is that?



- [ ] **A.** 25
- [ ] **B.** 24
- [ ] **C.** 20
- [ ] **D.** 30

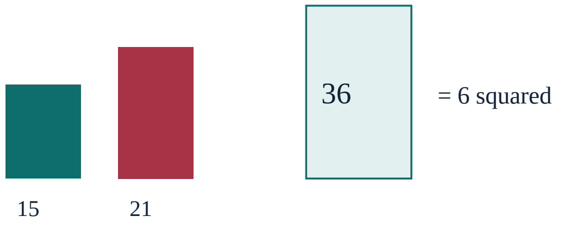
Q5. What formula gives the nth triangular number?

- [ ] **A.** n squared
- [ ] **B.** n(n+1)
- [ ] **C.** 2n
- [ ] **D.** n(n+1)/2

Q6. What is the 6th triangular number?

- [ ] **A.** 21
- [ ] **B.** 15
- [ ] **C.** 28
- [ ] **D.** 18

Q7. Read the two bars (15 and 21) and the tall combined bar. What total do they show?



- [ ] **A.** 35
- [ ] **B.** 40

- [ ] **C.** 36
- [ ] **D.** 30

Q8. What is the sum of the 1st triangular number (1) and the 2nd triangular number (3)?

- [ ] **A.** 3
- [ ] **B.** 4
- [ ] **C.** 5
- [ ] **D.** 6

Q9. What is the sum of the 2nd triangular number (3) and the 3rd triangular number (6)?

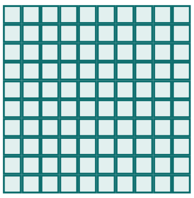
- [ ] **A.** 8
- [ ] **B.** 10
- [ ] **C.** 9
- [ ] **D.** 12

Q10. Compute 21 + 28, then check it against (6+1) squared. What is the sum of the 6th and 7th triangular numbers?

- [ ] **A.** 48
- [ ] **B.** 49
- [ ] **C.** 45
- [ ] **D.** 50

Section B - Medium

Q11. The 10-by-10 grid is (n+1) squared when n=9. What sum of consecutive triangular numbers is that?



n = 9, so n+1 = 10

10 by 10 square = 100

- [ ] **A.** 100
- [ ] **B.** 90
- [ ] **C.** 110
- [ ] **D.** 121

Q12. Which two consecutive triangular numbers sum to 100?

- ☐ **A.** 36 and 64
- ☐ **B.** 45 and 55
- ☐ **C.** 50 and 50
- ☐ **D.** 28 and 72

Q13. What is the 8th triangular number?

- ☐ **A.** 28
- ☐ **B.** 36
- ☐ **C.** 45
- ☐ **D.** 32

Q14. Follow the formula strip for the 10th and 11th triangular numbers. What product (the sum) is 11 x 11?

$n = 10$ + $n+1 = 11$ = 11×11

- ☐ **A.** 121
- ☐ **B.** 110
- ☐ **C.** 144
- ☐ **D.** 100

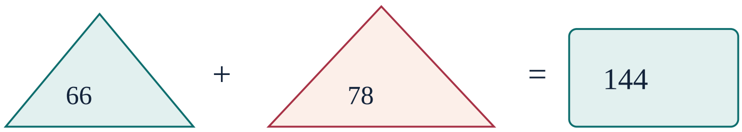
Q15. Which two consecutive triangular numbers sum to 49?

- ☐ **A.** 15 and 34
- ☐ **B.** 28 and 21
- ☐ **C.** 21 and 28
- ☐ **D.** 10 and 39

Q16. What is the 9th triangular number?

- ☐ **A.** 36
- ☐ **B.** 55
- ☐ **C.** 40
- ☐ **D.** 45

Q17. The two triangles are labelled 66 and 78. What do they add to?



- ☐ **A.** The 12th and 13th, 78 and 91
- ☐ **B.** The 10th and 11th, 55 and 66
- ☐ **C.** The 11th and 12th, 66 and 78
- ☐ **D.** The 144th triangular number alone

Q18. If (n+1) squared = 64, what pair of consecutive triangular numbers does this correspond to?

- ☐ **A.** The 8th and 9th triangular numbers, 36 and 45
- ☐ **B.** The 6th and 7th triangular numbers, 21 and 28
- ☐ **C.** The 64th triangular number alone
- ☐ **D.** The 7th and 8th triangular numbers, 28 and 36

Q19. What is the 10th triangular number?

- ☐ **A.** 55
- ☐ **B.** 45
- ☐ **C.** 66
- ☐ **D.** 50

Q20. Using the formula AND checking T(12)+T(13), what is the sum of the 12th and 13th triangular numbers?

- ☐ **A.** 169
- ☐ **B.** 196
- ☐ **C.** 225
- ☐ **D.** 156

Section C - Challenge

Q21. The two bars are the 19th and 20th triangular numbers. Which pair sums to 400?

- [ ] **A.** The 19th and 20th, 190 and 210
- [ ] **B.** The 20th and 21st, 210 and 231
- [ ] **C.** The 18th and 19th, 171 and 190
- [ ] **D.** The 400th triangular number alone

Q22. A student claims that adding any THREE consecutive triangular numbers (not just two) also always gives a perfect square. Test this on the 3rd, 4th and 5th triangular numbers: 6+10+15.

- [ ] **A.** True — 31 is a perfect square
- [ ] **B.** True, but only for the first three triangular numbers
- [ ] **C.** The sum cannot be computed
- [ ] **D.** False — 6+10+15=31, which is not a perfect square

Q23. What is the sum of the 24th and 25th triangular numbers?

- [ ] **A.** 600
- [ ] **B.** 576
- [ ] **C.** 625
- [ ] **D.** 650

Q24. The tilted square is 15 by 15. Which consecutive triangular pair sums to that 225?

- [ ] **A.** The 14th and 15th, 105 and 120
- [ ] **B.** The 15th and 16th, 120 and 136
- [ ] **C.** The 13th and 14th, 91 and 105
- [ ] **D.** The 225th triangular number alone

Q25. If two consecutive triangular numbers sum to a value that is NOT a perfect square, what must be true?

- ☐ **A.** The two numbers were not really triangular numbers
- ☐ **B.** This is common and expected
- ☐ **C.** An arithmetic mistake was made somewhere
- ☐ **D.** The rule only sometimes applies

Q26. What is the sum of the 49th and 50th triangular numbers?

- ☐ **A.** 2,450
- ☐ **B.** 2,500
- ☐ **C.** 2,401
- ☐ **D.** 2,550

Q27. The flow shows factoring T(n)+T(n+1). What is the factored form before (n+1) squared?

T(n)+T(n+1) → (n+1) x [n/2 + (n+2)/2]

- ☐ **A.** (n+1) x (n+2) x n / 2
- ☐ **B.** (n+1) x [n/2 + (n+2)/2]
- ☐ **C.** n squared plus (n+2) squared
- ☐ **D.** It cannot be factored at all

Q28. Anaya wants a pair of consecutive triangular numbers summing to 1,000. Does a whole-number pair exist?

- ☐ **A.** No — 1,000 is not a perfect square, so no such pair exists
- ☐ **B.** Yes — the 31st and 32nd triangular numbers work
- ☐ **C.** Yes — every whole number can be split this way
- ☐ **D.** Yes — the 999th and 1000th triangular numbers work

Q29. What is the sum of the 99th and 100th triangular numbers?

- ☐ **A.** 9,900
- ☐ **B.** 9,801
- ☐ **C.** 10,100

- [ ] **D.** 10,000

Q30. Why is proving the formula algebraically (as Anaya and Kabir did) more powerful than just checking a few examples?

- [ ] **A.** It only proves the pattern for small n
- [ ] **B.** It guarantees the pattern holds for every possible n, not just the cases that happened to be tested
- [ ] **C.** Checking examples is always more reliable than algebra
- [ ] **D.** Algebraic proof and checking examples give different answers

Answer Key

Q1: (A) (a) 16. 6 + 10 = 16, which is (3+1) squared = 4 squared.

Q2: (D) (d) (n+1) squared.

Q3: (B) (b) 15. Using n(n+1)/2 with n=5: 5 x 6 / 2 = 15.

Q4: (A) (a) 25. 10 + 15 = 25, which is (4+1) squared = 5 squared.

Q5: (D) (d) n(n+1)/2.

Q6: (A) (a) 21. Using n(n+1)/2 with n=6: 6 x 7 / 2 = 21.

Q7: (C) (c) 36. 15 + 21 = 36, which is (5+1) squared = 6 squared.

Q8: (B) (b) 4. 1 + 3 = 4, which is (1+1) squared = 2 squared.

Q9: (C) (c) 9. 3 + 6 = 9, which is (2+1) squared = 3 squared.

Q10: (B) (b) 49. 21 + 28 = 49, which is (6+1) squared = 7 squared.

Q11: (A) (a) 100. (n+1) squared with n=9 gives 10 squared = 100.

Q12: (B) (b) 45 and 55. The 9th triangular number is 45 and the 10th is 55.

Q13: (B) (b) 36. Using n(n+1)/2 with n=8: 8 x 9 / 2 = 36.

Q14: (A) (a) 121. (n+1) squared with n=10 gives 11 squared = 121.

Q15: (C) (c) 21 and 28. These are the 6th and 7th triangular numbers, matching n=6, (n+1) squared = 49.

Q16: (D) (d) 45. Using n(n+1)/2 with n=9: 9 x 10 / 2 = 45.

Q17: (C) (c) The 11th and 12th, 66 and 78. Since n+1=12, n=11, and 66+78=144.

Q18: (D) (d) The 7th and 8th triangular numbers, 28 and 36. Since n+1=8, n=7, and 28+36=64.

Q19: (A) (a) 55. Using n(n+1)/2 with n=10: 10 x 11 / 2 = 55.

Q20: (A) (a) 169. For n=12 the sum is (n+1) squared = 13 squared = 169. 196 would be 14 squared (the 13th and 14th).

Q21: (A) (a) The 19th and 20th, 190 and 210. Since n+1=20, n=19, and 190+210=400.

Q22: (D) (d) False — 6+10+15=31, which is not a perfect square. The perfect-square pattern is specific to summing exactly TWO consecutive triangular numbers.

Q23: (C) (c) 625. (n+1) squared with n=24 gives 25 squared = 625.

Q24: (A) (a) The 14th and 15th, 105 and 120. Since n+1=15, n=14, and 105+120=225.

Q25: (C) (c) An arithmetic mistake was made — this cannot happen for genuinely consecutive triangular numbers. The algebraic proof shows the sum is always exactly (n+1) squared, with no exceptions.

Q26: (B) (b) 2,500. (n+1) squared with n=49 gives 50 squared = 2,500.

Q27: (B) (b) (n+1) x [n/2 + (n+2)/2]. Factoring out the common (n+1) term from both triangular-number expressions is the key algebraic step.

Q28: (A) (a) No — 1,000 is not a perfect square, so no such pair exists. Only perfect squares can be reached, since the sum must equal (n+1) squared exactly.

Q29: (D) (d) 10,000. (n+1) squared with n=99 gives 100 squared = 10,000.

Q30: (B) (b) It guarantees the pattern holds for every possible n, not just the cases that happened to be tested. This is the core reason a general algebraic derivation is stronger evidence than a handful of matching examples.

You've got this! Keep learning and shining! - Ruchi Math Coaching

