The Sieve of Eratosthenes: Primes and Composites

Unit: Number Play - Student Practice Worksheet

Overview: Kabir tried to list every prime number from 1 to 100 by testing each one for factors, one at a time. Forty minutes in, he had checked eleven numbers and made three mistakes already. Ms. Rao handed him a grid and a pencil and said, "Cross out numbers instead of testing them. You'll be done in five minutes."

Practice Questions (30 Total)

Section A - Easy

Q1. The chips show every factor of 7. How many factors does this prime have?

- [ ] **A.** 2
- [ ] **B.** 1
- [ ] **C.** 3
- [ ] **D.** It varies

Q2. Is 1 a prime number?

- [ ] **A.** Yes
- [ ] **B.** No
- [ ] **C.** Only sometimes
- [ ] **D.** Only if doubled

Q3. Which of these is a prime number?

- [ ] **A.** 9
- [ ] **B.** 15
- [ ] **C.** 7
- [ ] **D.** 21

Q4. The strip shows what happens right after 2 is circled. What happens to the later evens?

2 4 6 8 10

After circling 2, every later even is crossed

- [ ] **A.** They stay circled as primes
- [ ] **B.** They are ignored
- [ ] **C.** They are divided by 3
- [ ] **D.** They are crossed out as multiples of 2

Q5. How many factors does a prime number have?

- [ ] **A.** Exactly two: 1 and itself
- [ ] **B.** Exactly one
- [ ] **C.** Three or more
- [ ] **D.** It varies with size

Q6. Which of these is a composite number?

- [ ] **A.** 13
- [ ] **B.** 12
- [ ] **C.** 11
- [ ] **D.** 17

Q7. The grey box is crossed out before any circling starts. Which special number is that?

~~1~~ then 2, 3, 4, 5...

- [ ] **A.** 0
- [ ] **B.** 2
- [ ] **C.** 1
- [ ] **D.** 100

Q8. Kabir wrongly called 21 a prime number. What was his mistake?

- ☐ **A.** 21 actually has no factors
- ☐ **B.** 21 is smaller than 20
- ☐ **C.** 21 is an even number
- ☐ **D.** He forgot 21 divides evenly by 7

Q9. What is the smallest prime number?

- ☐ **A.** 0
- ☐ **B.** 1
- ☐ **C.** 3
- ☐ **D.** 2

Q10. What is the name of the method that finds primes by crossing out multiples?

- ☐ **A.** The prime testing rule
- ☐ **B.** The Sieve of Eratosthenes
- ☐ **C.** The factor tree
- ☐ **D.** The number line method

Section B - Medium

Q11. After crossing multiples of 2 and 3, which remaining number in the strip is the next prime to circle?

2	3	4	5	6	7
8	9	10	11	12	13

- ☐ **A.** 5
- ☐ **B.** 9
- ☐ **C.** 4
- ☐ **D.** 12

Q12. Why does crossing out multiples of 2, then 3, then 5, then 7 already catch every composite number up to 100?

- ☐ **A.** Because 100 is itself prime

- ☐ **B.** Because only even numbers can be composite
- ☐ **C.** Because any composite up to 100 must have a prime factor of 10 or less
- ☐ **D.** It's a coincidence with no real reason

Q13. How many prime numbers are there between 1 and 20?

- ☐ **A.** 6
- ☐ **B.** 8
- ☐ **C.** 7
- ☐ **D.** 9

Q14. Among these four cards, which multiple of 3 would the sieve cross out first?

9	11	13	15

- ☐ **A.** 11
- ☐ **B.** 13
- ☐ **C.** 15
- ☐ **D.** 9

Q15. 37 survives every round of crossing out in a sieve of 1-50. What does this tell you?

- ☐ **A.** 37 must be prime
- ☐ **B.** 37 must be composite
- ☐ **C.** 37 must be equal to 1
- ☐ **D.** Nothing can be concluded

Q16. A number has factors 1, 2, 4, 8, and 16. Is it prime or composite?

- ☐ **A.** Prime, since it starts and ends with 1 and itself
- ☐ **B.** Composite, since it has more than two factors
- ☐ **C.** Neither, since it's a power of 2
- ☐ **D.** Cannot be determined

Q17. Which pair of circled numbers on this strip are both prime?

15 16 17 18 19 20

- A. 15 and 16
- B. 18 and 20
- C. 17 and 19
- D. 16 and 18

Q18. In the sieve method, once you've crossed out multiples of 2, 3, 5, and 7 up to 100, do you need to check multiples of 11 too?

- A. Yes, always
- B. Only for odd numbers
- C. Only if 11 is composite
- D. No, since 11×11 exceeds 100

Q19. Which of these numbers is composite but NOT a multiple of 2 or 3?

- A. 14
- B. 27
- C. 25
- D. 18

Q20. How many composite numbers are there between 2 and 10 inclusive?

- A. 3
- B. 5
- C. 4
- D. 6

Section C - Challenge

Q21. On this 90s strip, which single circled number survives a full sieve?

91 93 95 97 99

91=7×13, 93=3×31, 95=5×19, 99=9×11

- [ ] **A.** 97
- [ ] **B.** 93
- [ ] **C.** 99
- [ ] **D.** 91

Q22. Which statement correctly compares the sieve method to Kabir's original one-by-one testing approach?

- [ ] **A.** Both methods are equally fast
- [ ] **B.** The sieve avoids repeated division testing by reusing earlier crossing-out work
- [ ] **C.** One-by-one testing is always more accurate
- [ ] **D.** The sieve only works for numbers under 20

Q23. A sieve up to 30 crosses out 1, then multiples of 2, 3, and 5. Which number is the LARGEST that survives?

- [ ] **A.** 27
- [ ] **B.** 29
- [ ] **C.** 28
- [ ] **D.** 25

Q24. When sieving multiples of 5, why does the highlighted 25 come before any still-open multiple of 5?

10 15 20 25

10, 15, 20 were already multiples of 2 or 3

- [ ] **A.** Because 10 is not a real number
- [ ] **B.** Because 5 has no smaller multiples
- [ ] **C.** Because 10, 15, 20 were already crossed out as multiples of 2 or 3
- [ ] **D.** There is no real reason — starting point doesn't matter

Q25. A number less than 100 has exactly 3 distinct prime factors. Can it still be found by a standard sieve of 1-100?

- ☐ **A.** No, the sieve only works for numbers with 2 prime factors
- ☐ **B.** No, such numbers are always prime
- ☐ **C.** Only if all three factors are the same
- ☐ **D.** Yes — the sieve marks ALL composites, regardless of how many distinct prime factors they have

Q26. In a sieve of 1-100, which round of crossing-out eliminates the number 91?

- ☐ **A.** The multiples-of-7 round, since 91=7×13
- ☐ **B.** The multiples-of-2 round
- ☐ **C.** The multiples-of-3 round
- ☐ **D.** 91 is never eliminated — it's prime

Q27. If the sieve skips the multiples-of-3 round, what happens to the peach cells?

Peach cells would stay wrongly uncrossed

- ☐ **A.** Real primes like 7 get crossed out by mistake
- ☐ **B.** The number 11 becomes composite
- ☐ **C.** Composites like 9 and 21 stay wrongly uncrossed as if prime
- ☐ **D.** Nothing changes at all

Q28. A student says the sieve proves there are only a limited number of primes, since eventually everything gets crossed out. Is this reasoning sound?

- ☐ **A.** Yes, the sieve proves primes are finite
- ☐ **B.** Yes, but only for even limits
- ☐ **C.** The sieve is unrelated to primes
- ☐ **D.** No, the sieve just has a chosen limit

Q29. Anaya wants to extend a sieve from 1-100 to 1-200. Up to which prime must she now cross out multiples, to guarantee every composite under 200 is caught?

- ☐ **A.** 7, same as before
- ☐ **B.** 13, since its square is just under 200

- [ ] **C.** 19, to be extra safe
- [ ] **D.** 100, half of 200

Q30. Kabir's one-by-one testing missed 21 as composite. Which sieve flaw would cause a similar miss?

- [ ] **A.** Stopping the crossing-out rounds too early, before reaching a prime whose square is still under the limit
- [ ] **B.** The sieve has no possible flaw ever
- [ ] **C.** Crossing out too many numbers by mistake
- [ ] **D.** Starting the sieve at 0 instead of 1

Answer Key

Q1: (A) (a) 2. The diagram has exactly two chips, 1 and 7 — that is the definition of a prime.

Q2: (B) (b) No. 1 has only one factor, itself, so it is neither prime nor composite.

Q3: (C) (c) 7. 7's only factors are 1 and 7. The others all have more than two factors — 9=3×3, 15=3×5, 21=3×7.

Q4: (D) (d) They are crossed out as multiples of 2. Circling 2 marks it prime; every later even is a multiple and must go.

Q5: (A) (a) Exactly two: 1 and itself. More than two factors would make the number composite.

Q6: (B) (b) 12. 12 has factors 1, 2, 3, 4, 6, and 12 — more than two. The others are all prime.

Q7: (C) (c) 1. Since 1 is neither prime nor composite, it is crossed out at the very start.

Q8: (D) (d) He forgot 21 divides evenly by 7. 21=3×7, giving it more than two factors, so it is composite.

Q9: (D) (d) 2. 2 is the smallest, and only even, prime — every other even number is divisible by 2.

Q10: (B) (b) The Sieve of Eratosthenes. Named after the ancient Greek mathematician who devised this crossing-out method.

Q11: (A) (a) 5. 4, 6, 8, 9, 10, 12 are already crossed. 5 is the smallest uncrossed number after 2 and 3.

Q12: (C) (c) Because any composite up to 100 must have a prime factor of 10 or less. If both factors exceeded 10, the product would exceed 100, so sieving through 7 is enough.

Q13: (B) (b) 8. The primes up to 20 are 2, 3, 5, 7, 11, 13, 17, 19.

Q14: (D) (d) 9. The peach card 9 is the smallest multiple of 3 in the row, so it is crossed before 15.

Q15: (A) (a) 37 must be prime. Surviving every round of crossing-out is exactly the sieve's definition of being prime.

Q16: (B) (b) Composite, since it has more than two factors. Five factors automatically disqualifies it from being prime.

Q17: (C) (c) 17 and 19. Both are circled and uncrossed. Every other pair on the strip is already struck through.

Q18: (D) (d) No, since 11×11 exceeds 100. Any composite multiple of 11 under 100 already has a second factor of 9 or less, caught earlier.

Q19: (C) (c) 25. 25=5×5 is composite but not divisible by 2 or 3 — it only gets crossed out in the multiples-of-5 round.

Q20: (B) (b) 5. From 2 to 10, the composites are 4, 6, 8, 9, and 10, while 2, 3, 5, 7 are prime.

Q21: (A) (a) 97. The caption already factors the struck numbers: 91=7×13, 93=3×31, 95=5×19, 99=9×11.

Q22: (B) (b) The sieve avoids repeated division testing by reusing earlier crossing-out work. Once a number is crossed out, it never needs to be tested again.

Q23: (B) (b) 29. 27=3×9 and 25=5×5 get crossed out, 28 is even, and 29 has no factors besides 1 and itself.

Q24: (C) (c) Because 10, 15, 20 were already crossed out as multiples of 2 or 3. 25 is the first multiple of 5 with no smaller prime factor.

Q25: (D) (d) Yes — the sieve marks ALL composites, regardless of how many distinct prime factors they have.

Q26: (A) (a) The multiples-of-7 round, since 91=7×13. 91 is odd and not divisible by 3 or 5, so it survives until 7.

Q27: (C) (c) Composites like 9 and 21 stay wrongly uncrossed as if prime. Skipping a round never falsely crosses out a real prime.

Q28: (D) (d) No, the sieve just has a chosen limit. Finding a fixed count in 1-100 says nothing about primes beyond 100.

Q29: (B) (b) 13, since its square is just under 200. 13×13=169 is under 200, but 17×17=289 exceeds it.

Q30: (A) (a) Stopping the crossing-out rounds too early. If you stop after only 2 and 3, composites like 25 or 49 would wrongly survive.

You've got this! Keep learning and shining! - Ruchi Math Coaching